Empirical Modelling of Japan's Markup and Inflation, 1976-2000

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Abstract

This paper aims to pursue an empirical model for Japan's markup and inflation using historical time series data covering the last quarter of the 20th century. A multivariate cointegration analysis of Japan's macroeconomic data indicates the existence of a long-run economic linkage, which is interpreted as an empirical representation of countercyclical markup. A set of variables in the cointegrated system, apart from markup and inflation, are judged to be weakly exogenous for parameters of interest, thereby allowing us to estimate a partial model given the weakly exogenous variables. The model reduction is then conducted to achieve a parsimonious representation of countercyclical markup and inflation dynamics over the sample period of interest.

KEY WORDS: Countercyclical Markup, Inflation Dynamics, Cointegration, Vector Autoregressive Model, Weak Exogeneity.

1 Introduction

The objective of this paper is to model Japan's markup and inflation using historical time series data covering the last quarter of the 20th century. It is demonstrated that a cointegrated vector autoregressive (VAR) model reduces to a parsimonious data-congruent dynamic system subject to economic interpretations. The introductory section briefly reviews the related literature and describes the most significant aspects of the present paper.

Interaction between prices and wages has been of great interest for economists, and theoretical models for prices and wages have played a key role in the development of modern macroeconomics (see Romer, 2001, Ch.5, *inter alia*). Tobin (1972) presents a two-dimensional theoretical model encompassing a *markup* equation, and the model is successful in describing dynamic interdependent relationships of prices and wages. See also Blanchard and Fisher (1989, Ch.10) for Tobin's model. Regarding applied work using

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time series data, Sargan (1964) is a seminal paper in that he gives congruent econometric representations of wages and prices in the UK based on an equilibrium correction approach, the approach intimately linked to a cointegration analysis. A recent empirical analysis of price and wage time series data was performed by Bårdsen, Jansen and Nymoen (2003), Marcellino and Mizon (2001), *inter alia*.

Macroeconomic time series data often exhibit non-stationary behaviour, and thus need to be treated as integrated processes rather than stationary. The concept of cointegration introduced by Granger (1981) therefore plays an important role in time series econometrics, and a cointegrated VAR model developed by Johansen (1988, 1996) has become a major econometric tool for macro and financial economists. See Juselius (2006) for extensive empirical research using a cointegrated VAR model. The cointegrated VAR analysis is well fitted in general-to-specific modelling approach, due to the fact that the analysis usually starts with the investigation of general unrestricted VAR models (see Campos, Ericsson and Hendry, 2006, for details of the approach). Hendry and Mizon (1993) discuss a model reduction procedure using the cointegrated VAR model. See also Hendry and Doornik (1994), Kurita (2007) for the general-to-specific modelling methodology using the cointegrated VAR analysis. Furthermore, the concept of weak exogeneity introduced by Engle, Hendry and Richard (1983) also plays an important part in econometrics. Weak exogeneity permits us to model a partial or conditional system alone, instead of a full system, for the purpose of making efficient statistical inferences on parameters of interest. Weak exogeneity in the cointegrated VAR system is explored by Johansen (1992) and Urbain (1992). Cointegration and weak exogeneity provide a methodological basis for empirical investigation pursued in this paper.

This paper aims to achieve a data-congruent representation of markup and inflation using Japan's historical time series data. The empirical exploration sheds useful light on deeper understanding of the Japanese economy in 1976 - 2000, a quarter-century period of Japan's economic turmoil, during which an asset-price bubble economy took place and then collapsed, leading to Japan's lost decade. See Yoshikawa (2002) for details of the Japanese economy during the lost decade. This paper adopts a cointegrated VAR approach to modelling the data of markup, inflation and various other macroeconomic series in Japan. The analysis indicates the existence of a long-run economic linkage interpreted as an empirical representation of *countercyclical markup*, see Rotemberg and Woodford (1999), *inter alia*. A set of variables in the cointegrated system, apart from markup and inflation, are judged to be weakly exogenous for parameters of interest, thereby allowing us to estimate a partial model given the weakly exogenous variables. The model reduction is then conducted so as to achieve a parsimonious dynamic econometric system for markup and inflation. To the best of the author's knowledge, the present paper is the first empirical study that is successful in achieving a data-congruent representation of these two variables for 1976 - 2000, corresponding to the period of the pre and post Japan's bubble economy. It is noteworthy that such a stable structure has been revealed from the analysis of the data covering the period of Japan's economic turmoil.

The organization of this paper is as follows. Section 2 reviews countercyclical markup and inflation dynamics. Section 3 pursues econometric modelling of markup and inflation using the cointegrated VAR approach. Section 4 provides the overall summary and conclusion. Details of the data to be analysed are provided in the Appendix. All the empirical analysis and graphics in this paper use *OxMetrics / PcGive* (Doornik and Hendry, 2006).

2 Countercyclical Markup and Inflation Dynamics

This section introduces a set of economic variables to be analysed in a cointegrated VAR model. Since the aim of this paper is to estimate an empirical model for Japan's markup and inflation, it is necessary to conceive a plausible long-run economic relationship associated with these two variables — an interpretable economic linkage which may correspond to an empirical cointegrating relation estimated from the data. To this end, let us suppose that markup pricing is formulated as

$$P_t = \theta_t \left(\frac{W_t}{A_t} \right), \tag{1}$$

where P_t is the price level, W_t is the nominal wage, A_t is the labour productivity, and θ_t is the markup. Let us introduce the output Y_t and define $y_t = \log Y_t$. In order to map (1) to an observable relationship subject to an empirical investigation, the following specifications of A_t and θ_t are assumed:

$$\log \theta_t = -\delta \Delta y_t + u_t \quad \text{and} \quad \log A_t = \eta t, \tag{2}$$

where u_t represents a stationary error term capturing unspecified dynamics. The specification of θ_t is based on the fact that the markup tends to be moderately *countercyclical* with economic growth; see Blanchard and Fisher (1989, Ch.9), Solon, Barsky and Parker (1994), Rotemberg and Woodford (1999), Romer (2001, Ch.5), *inter alia*. Various theories have been developed to explain the countercyclical behaviour of markup such as collusion in imperfect competition and kinked demand curves; see the references above. The productivity A_t is specified to be an exponential function of deterministic trend in (2), based on the assumption that productivity growth should be approximated to a stable upward trending path. Hence substituting (2) into (1) and taking logs of both sides leads to

$$p_t - (w_t - \eta t) + \delta \Delta y_t = u_t, \tag{3}$$

for $p_t = \log P_t$ and $w_t = \log W_t$. Equation (3) is a candidate for a cointegrating relationship based on the notion of countercyclical markup. Let us define productivity-adjusted wage $w_t^* = w_t - \eta t$ for future reference. An inflation process should be dependent on the markup, possibly adjusting to disequilibrium errors represented by (3). It is therefore reasonable to conceive the following bivariate equilibrium correction model (ECM) capturing countercyclical markup and inflation dynamics simultaneously:

$$\begin{aligned} \begin{bmatrix} \Delta (p_t - w_t^*) \\ \Delta^2 p_t \end{bmatrix} &= \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} + \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \left[p_{t-1} - w_{t-1}^* + \delta \Delta y_{t-1} \right] \\ &\quad + \sum_{i=1}^{l-1} \begin{bmatrix} \gamma_{11,i} & \gamma_{12,i} \\ \gamma_{21,i} & \gamma_{22,i} \end{bmatrix} \begin{bmatrix} \Delta (p_{t-} - w_t^*) \\ \Delta^2 p_t \end{bmatrix} + \begin{bmatrix} \nu_{1,t} \\ \nu_{2,t} \end{bmatrix}, \end{aligned} \tag{4}$$

where $\nu_{j,t}$ for $j = 1, 2$ is a set of stationary processes consisting of omitted short-run dynamic terms and innovations. Such a bivariate system as (4) may be seen as an empirical representation of Tobin's wage-price model (Tobin, 1972). This paper is interested in estimating an empirical dynamic model for markup and inflation, thus the ECM given by (4) is seen as a target representation which should be achieved as result of econometric investigation. Furthermore, it is often pointed out that the spread between the long and short term interest rates contains information about expected future economic growth. See Stock and Watson (1989), Bernard and Gerlach (1998), Hamilton and Kim (2002), and Ichiue (2004), *inter alia*. The interest rate differential or yield spread, denoted by rs_t, may play a significant role in the short-run dynamics, or $\nu_{i,t}$, of the ECM.

The argument so far allows us to introduce an $I(1)$ cointegrated VAR(k) model encompassing (4), formulated as

$$X_t = (p_t - w_t, \Delta p_t, \Delta y_t, rs_t)', \tag{5}$$

and

$$\Delta X_t = \alpha\,(\beta', \gamma) \begin{pmatrix} X_{t-1} \\ t \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu + \varepsilon_t, \quad \text{for} \quad t = 1, ..., T, \tag{6}$$

where a sequence of innovations ε_t has independent and identical normal $N(0, \Omega)$ distributions conditional on $X_{-k+1}, ..., X_0$, and $\alpha, \beta \in \mathbf{R}^{4\times r}$ for $r < 4$, $\gamma \in \mathbf{R}^{r\times 1}$, $\mu \in \mathbf{R}^{4\times 1}$ and $\Gamma_i \in \mathbf{R}^{4\times 4}$. Let $\beta^{*\prime} = (\beta', \gamma)$ and $X^*_{t-1} = \left(X'_{t-1}, t\right)'$ for future reference. Johansen (1996) demonstrates details of likelihood-based inference for these parameters. In equation (6) α is referred to as adjustment vectors, while $\beta^{*\prime}$ is called cointegrating vectors. In practice, there may be a case where both p_t and w_t exhibit $I(2)$-type non-stationary behaviour (see Juselius, 2006, Ch.16, *inter alia*). Considering the presence of a markup relation between p_t and w_t, it is reasonable to conjecture that $p_t - w_t$ is seen as an $I(1)$ process as a result of the removal of the common $I(2)$ stochastic trend or nominal-to-real transformation (see Kongsted, 2005). This conjecture is confirmed by an overview of time series plots of $p_t - w_t$ in the next section. In addition, there is a possibility that Δy_t may be seen as a stationary process rather than $I(1)$. As shown in Johansen (1996, page 74), it is possible to include stationary variables in model (6) as long as they are relevant in terms of economic theory and insight. In the empirical analysis performed in this paper, it is checked whether Δy_t is judged to be either a stationary process or an $I(1)$ process.

As the cointegrating rank r is unknown in practice, the rank needs to be determined based on the data analysis. A log-likelihood ratio ($\log LR$) test statistic consists of the null hypothesis of r cointegration rank $\mathsf{H}\,(r)$ against the alternative hypothesis $\mathsf{H}\,(p)$, and its asymptotic quantiles are provided by Johansen (1996, Ch.15). Determining the cointegrating rank in (6) allows us to test various restrictions on α, β and γ in order to pursue the adjustment structure and cointegrating relationships subject to economic interpretation. Cointegrating relationships, embodied by $\beta^{*\prime} X^*_{t-1}$, correspond to a set of stationary linear combinations, acting as equilibrium correction mechanisms in (6). Thus it is necessary to check empirically if (3) belongs to a class of cointegrating relationships.

Next, in order to derive (4) from (6) as a partial or conditional data-representation, let the process be decomposed as $X_t = (X'_{1t}, X'_{2t})'$ for $X_{1t} = (p_t - w_t, \Delta p_t)'$ and $X_{2t} =$

$(\Delta y_t, rs_t)'$. The parameters and error terms appearing in (6) are also expressed as

$$\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}, \ \Gamma_i = \begin{pmatrix} \Gamma_{1,i} \\ \Gamma_{2,i} \end{pmatrix}, \ \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \ \varepsilon_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}, \ \Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}.$$

We are interested in estimating (4), or a bivariate system for X_{1t} conditional on X_{2t} with no loss of information. Suppose $\alpha_2 = 0$, then (6) is decomposed into a model for X_{1t} conditional on X_{2t} and a marginal model for X_{2t} as follows:

$$\Delta X_{1t} = \omega \Delta X_{2t} + \alpha_1 \beta^{*\prime} X^*_{t-1} + \sum_{i=1}^{k-1} \widetilde{\Gamma}_{1,i} \Delta X_{t-i} + \widetilde{\mu}_1 + \widetilde{\varepsilon}_{1,t}, \tag{7}$$

$$\Delta X_{2t} = \sum_{i=1}^{k-1} \Gamma_{2,i} \Delta X_{t-i} + \mu_2 + \varepsilon_{2,t}, \tag{8}$$

where

$$\omega = \Omega_{12}\Omega_{22}^{-1}, \ \widetilde{\Gamma}_{1,i} = \Gamma_{1,i} - \omega \Gamma_{2,i}, \ \widetilde{\mu}_1 = \mu_1 - \omega \mu_2, \ \widetilde{\varepsilon}_{1,t} = \varepsilon_{1,t} - \omega \varepsilon_{2,t},$$

and

$$\begin{pmatrix} \widetilde{\varepsilon}_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} = N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Omega_{11.2} & 0 \\ 0 & \Omega_{22} \end{pmatrix} \right],$$

for

$$\Omega_{11.2} = \Omega_{11} - \Omega_{12}\Omega_{22}^{-1}\Omega_{21}.$$

Note that $\beta^{*\prime} X^*_{t-1}$ is not embedded in the marginal model (8). It is then possible to say that (4) may correspond to the conditional model (7). Under the condition of $\alpha_2 = 0$, X_{2t} is said to be weakly exogenous for the following parameters of interest:

$$\alpha_1, \ \beta^*, \ \omega, \ \widetilde{\Gamma}_{1,1}, ..., \widetilde{\Gamma}_{1,k-1}, \ \widetilde{\mu}_1, \text{ and } \Omega_{11.2}. \tag{9}$$

The parameters in (9) correspond to those appearing in (4), and may therefore be treated as the set of parameters of interest. It is true that X_{1t} may cause X_{2t} in the sense of Granger (1969) by way of its short-run dynamics in the marginal model (8), *e.g.* the past values of $\Delta(p_t - w_t^*)$ and $\Delta^2 p_t$ are likely to have influences on Δrs_t in (8). However, as long as the condition for weak exogeneity $\alpha_2 = 0$ is satisfied, the parameters of interest (9) can be estimated from the conditional model (7) alone without loss of information, with no need for the estimation of the marginal model (8). See the Appendix of this paper and Johansen (1992) for further details of a partial system and weak exogeneity.

3 Empirical Analysis of Markup and Inflation

This section conducts an econometric analysis of Japan's historical time series data, which aims at achieving a data-congruent parsimonious representation of markup and inflation. Section 3.1 presents an overview of the data in question, and Section 3.2 discusses misspecification tests of the initial unrestricted VAR model. Sections 3.3 and 3.4 perform a comprehensive $I(1)$ cointegrated VAR analysis. The cointegrating rank is investigated in Section 3.3, and the identification of long-run economic relationships and weakly exogenous variables is explored in Section 3.4. Finally, Section 3.5 achieves a parsimonious equilibrium correction system for markup and inflation, checking the model's adequacy and pursuing its economic interpretation.

3.1 An Overview of the Data

An overview of Japan's time series data for X_t defined in (5) is presented in this subsection. See the Appendix for details of the data. This paper is interested in modelling Japan's historical data for the last quarter of the 20th century, covering the period of the pre and post bubble economy. The data of Japan's GDP based on a previous version of the system of national accounts, known as 68SNA, are available for the whole sample period of interest, and therefore suitable for the analysis pursued in this paper. The period of the data given in Figure 1 runs from the third quarter in 1975 to the final quarter in 2000, denoted by 1975.3 - 2000.4 hereafter.

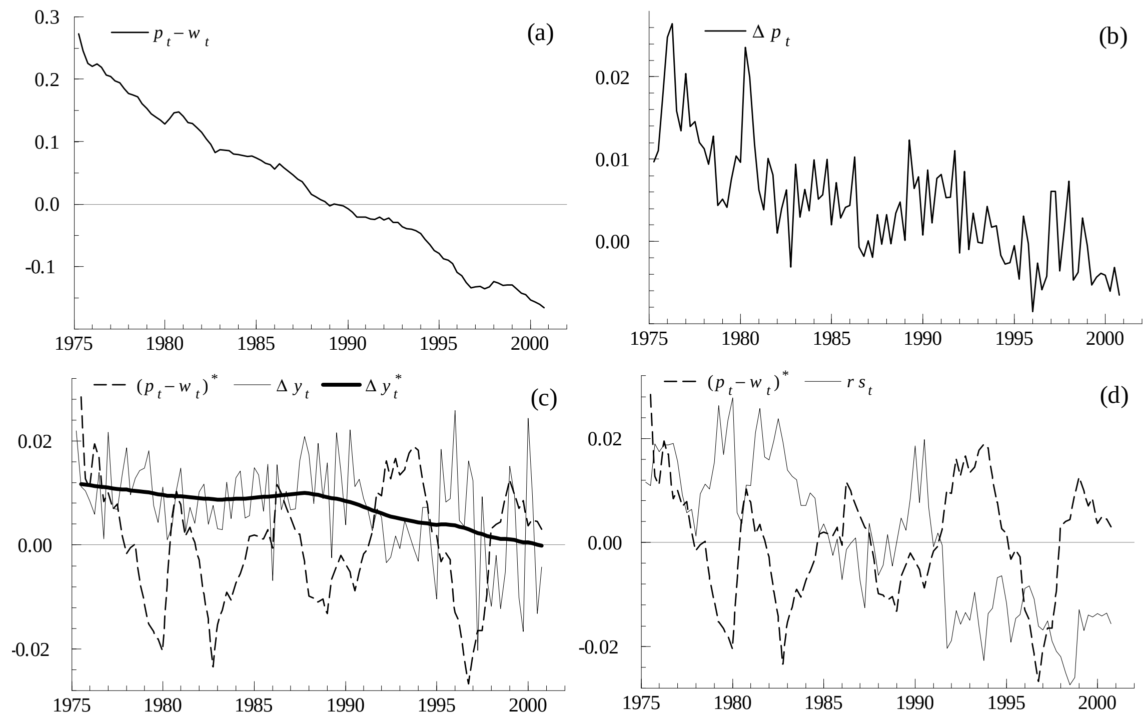

Figure 1: An Overview of Japan's Macroeconomic Data

Figure 1(a) presents time series data of $p_t - w_t$. The data wander around deterministic trend and seem to be free from any $I(2)$-type features, probably due to the removal of the common $I(2)$ trend from p_t and w_t, as discussed in the previous section. Thus $p_t - w_t$ is seen as an $I(1)$ process incorporating deterministic trend, which is interpreted as a local approximation to productivity, as discussed in the last section. Next, Figure 1(b) displays data plots of Δp_t. This variable appears to be subject to stochastic trend, so it can be an $I(1)$ series rather than stationary. This finding is also consistent with the view that p_t in level is often found to be an $I(2)$ process in the literature, as discussed in the last section.

Next, Figure 1(c) displays plots of residuals from the regression of $p_t - w_t$ on linear trend and constant, together with those of Δy_t. The figure also presents plots of Δy_t^*, which denotes the smoothed level of Δy_t using the Kalman filter (see Koopman, Harvey, Doornik and Shephard, 2006). The residual plots, denoted by $(p_t - w_t)^*$, represent trend-adjusted markup. It is anticipated that, as a result of the elimination of deterministic trend by running the regression, the trend-adjusted markup exhibits countercyclical features. Figure 1(c) seems to indicate some evidence for negative correlation between $(p_t - w_t)^*$ and Δy_t, in line with equation (3). In addition, Δy_t^* exhibits downward trending behaviour, indicating the possibility that Δy_t should be treated as a non-stationary series rather than stationary. Whether countercyclical markup given by (3) leads to a valid empirical representation or not is a question to be answered using a thorough cointegration analysis, together with the question of whether Δy_t is judged to be a stationary or non-stationary series.

Figure 1(d) displays the data plots of $(p_t - w_t)^*$ as well as those of rs_t. The scale of the vertical axis is normalised for $(p_t - w_t)^*$. The figure also indicates the presence of negative correlation between $(p_t - w_t)^*$ and rs_t, in line with the view of countercyclical markup. It is of interest, in the following cointegration analysis, to check whether either Δy_t or rs_t is relevant in an empirical long-run economic linkage corresponding countercyclical markup.

Finally, let us note that no figure indicates any clear evidence for influences of the Asian currency crisis happening in the Autumn of 1997. This is not surprising because the Japanese yen has been floating since the breakdown of the Bretton Woods regime in the early 1970s and was not directly involved in the currency crisis. As far as modelling Japan's markup and inflation is concerned, the Asian crisis seems to play a limited role and it is probably not necessary to pay any special attention to the event in this paper.

3.2 Estimating the Unrestricted VAR Model

The empirical analysis commences with a general unrestricted VAR(5) model incorporating two deterministic terms, a constant and linear trend. The inclusion of deterministic trend can be justified as it is treated as a local approximation to productivity; see the argument in Section 2. F-tests available in *PcGive* indicate that variables at lag length 5 seem to be irrelevant so the VAR(5) model reduces to the VAR(4) model. Some evidence is then found for significance of a variable with lag length 4, suggesting that further model reduction is likely to be inappropriate. This conjecture is also supported by the Akaike information criteria (AIC) when the sample period is adjusted according to lag length; the AIC of VAR(5), VAR(4) and VAR(3) models are -22.32, -22.42 and -22.38, respectively. The AIC of the VAR(4) model is the smallest one and seems to be in favour of this model. Thus the overall evidence allows us to choose the VAR(4) model, so that the sample period for estimation is 1976.3 - 2000.4 and the number of observations is 98.

The unrestricted VAR model is a purely statistical representation, so the estimated coefficients are not necessarily subject to economic interpretation. After identifying the cointegrating relations and conducting the model reduction, it is possible to pursue such interpretation. The unrestricted VAR model provides a starting point towards a parsimonious representation, and should therefore pass a set of residual diagnostic tests such as non-autocorrelation and normality.

The diagnostic tests of the VAR(4) model are given in Table 1. Most of the test results are given in the form $\mathrm{F}_j(k, T-l)$, which means an approximate F-test against the alternative hypothesis j : kth-order serial correlation (F_{ar}: see Godfrey, 1978; Nielsen,

Single equation tests	$p_t - w_t$	Δp_t	Δy_t	rs_t
Autocorr. [$F_{ar}(6,74)$]	1.48 [0.20]	1.47 [0.20]	0.50 [0.81]	1.37 [0.24]
ARCH [$F_{arch}(6,68)$]	0.18 [0.98]	0.09 [1.00]	1.09 [0.38]	0.79 [0.58]
Hetero.[$F_{het}(34,45)$]	0.61 [0.93]	0.72 [0.84]	0.87 [0.67]	0.49 [0.98]
Normality [$\chi^2_{nd}(2)$]	3.16 [0.21]	3.02 [0.22]	3.66 [0.16]	2.22 [0.33]

Vector tests			
Autocorr.[$F_{ar}(16,223)$]	1.22 [0.25]	Hetero.[$F_{het}(340,374)$]	0.66 [1.00]
- [$F_{ar}(96,212)$]	1.04 [0.41]	Normality [$\chi^2_{nd}(8)$]	7.17 [0.52]

Note. Figures in brackets are p-values.

Table 1: Mis-Specification Tests for the Unrestricted VAR Model

2007), kth-order autoregressive conditional heteroscedasticity (F_{arch}: see Engle, 1982), heteroscedasticity (F_{het}: see White, 1980), and a chi squared test for normality (χ^2_{nd}: see Doornik and Hansen, 1994). The diagnostic test statistics are all insignificant, thereby allowing us to conclude that this model is subject to the subsequent likelihood-based cointegration analysis and model reduction.

3.3 Choosing the Cointegrating Rank

This sub-section determines the cointegrating rank r in the VAR model. Table 2 presents two types of $\log LR$ test statistics for the choice of r, so-called trace test statistics (*trace*) and maximum eigenvalue test statistics (*max.eigen.*). The modulus of the six largest roots of a companion matrix for the VAR model are also provided in the table.

	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$
trace	64.92[0.04]*	40.51[0.08]	23.77[0.09]	9.66[0.15]
max.eigen.	24.41[0.32]	16.73[0.48]	14.12[0.25]	9.66[0.15]

mod ($r = 1$)	1.00	1.00	1.00	0.78	0.78	0.76
mod ($r = 2$)	1.00	1.00	0.83	0.83	0.79	0.78

Note. * denotes significance at the 5% level.

Table 2: Determination of the Cointegration Rank

According to the first panel, the trace test rejects the null hypothesis of $r = 0$ and does not reject the remaining ones at the 5% significance level, hence supporting $r = 1$. In contrast, the maximum eigenvalue test does not reject any null hypotheses at the same level. As discussed by Juselius (2006, Ch.8), the choice of the cointegrating rank is a very difficult task and so we should make use of as much additional information as possible

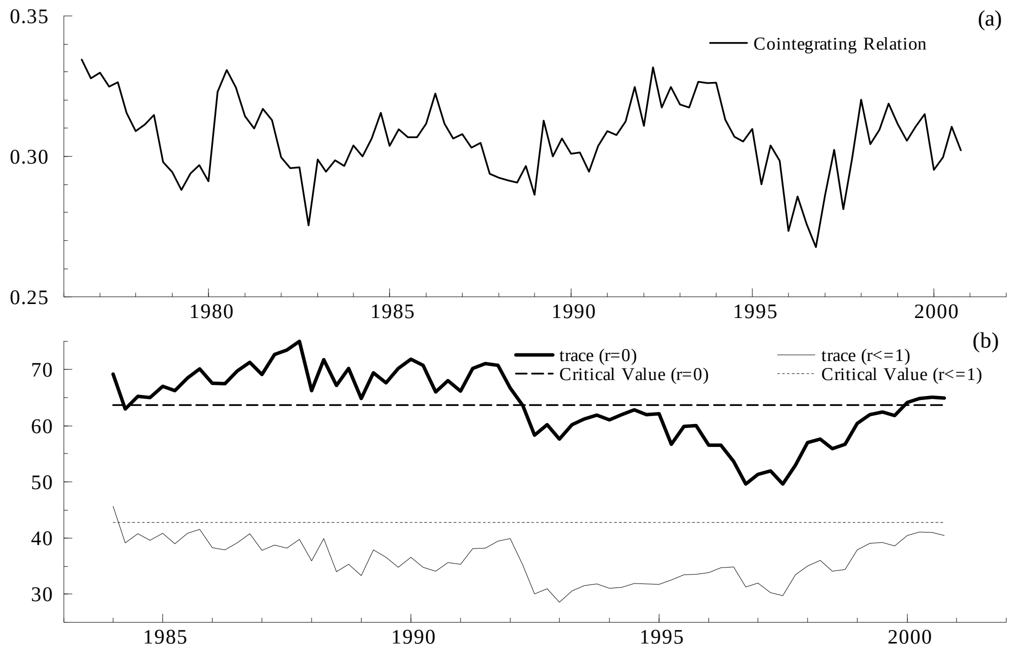

Figure 2: Cointegrating Relation and Recursive Trace Test Statistics

in order to determine the rank. The second panel, motivated by the results of the trace test in the first panel, provides modulus (denoted mod) of the six largest eigenvalues of a companion matrix of the VAR model restricted with $r = 1$ or with $r = 2$. No eigenvalue over 1.0 suggests that the model does not include any explosive root, and all the eigenvalues apart from the imposed unit roots are distinct from unity. Judging from these outcomes, the restriction of $r = 1$ seems to be appropriate for the description of the data.

In order to consolidate the argument for $r = 1$, Figure 2(a) presents time series plots of the estimated cointegrating combination under the restriction of $r = 1$, and Figure 2(b) displays recursive plots of some of the trace test statistics. According to Figure 2(a), the cointegrating relation exhibits no trending behaviour and its variance appears to be stable over the sample period. This feature is seen as evidence for stationarity, that is, evidence against the restriction of $r = 0$, hence supporting the presence of cointegration in the data. Two recursive trace test statistics presented in Figure 2(b) are for the cases of $r = 0$ and $r \leq 1$ respectively, and the corresponding critical values are displayed by thick and thin dotted lines. The first trace test rejects the null hypothesis $r = 0$ in the first half of the sample period for recursive estimation, then getting a bit smaller than the critical value and again rejecting the null hypothesis at the end of the sample, which is in line with the result of the trace test in Table 2. In contrast, the second trace test does not reject the null hypothesis $r \leq 1$ apart from the beginning of recursive estimation. The evidence in Figure 2(b) is in favour of the restriction of $r = 1$ rather than $r = 2$. The overall argument thus far allows us to conclude $r = 1$, and the empirical exploration is continued with the restriction of $r = 1$.

	$p_t - w_t$	Δp_t	Δy_t	rs_t
$\log LR$	$20.70[0.00]^{**}$	$15.01[0.00]^{**}$	$16.94[0.00]^{**}$	$21.58[0.00]^{**}$

Note. Figures in the square brackets are p-values according to $\chi^2(4)$

Table 3: Testing Stationarity of Each Variable in the System

Next, using a testing procedure suggested by Johansen (1996, p.74), we also test whether each variable is judged to be individually stationary. This test is to check if the cointegrating vector can be expressed as a unit vector consisting of unity and zeros, *e.g.* $\beta^{*\prime} = (1, 0, 0, 0, 0)$, such that the variable corresponding to unity in $\beta^{*\prime}$ is individually stationary. This test therefore corresponds to a unit root test in the VAR framework. The hypothesis of stationarity is rejected with respect to each variable in the system at the 5% significance level as shown in Table 3, suggesting that all the variables need to be treated as non-stationary series.

3.4 Identifying the Long-Run Economic Relationship

This sub-section explores valid restrictions on the adjustment and cointegrating space in the $I(1)$ cointegrated VAR system. The determination of the cointegrating rank, or $r = 1$, enables us to inspect such restrictions using a standard χ^2-based asymptotic inference.

	$p_t - w_t$	Δp_t	Δy_t	rs_t	t
$\widehat{\alpha}'$	-0.11	-0.15	1.93	0.19	$-$
	(0.04)	(0.03)	(4.06)	(0.07)	
$\widehat{\beta}^{*\prime}$	1.00	1.10	2.64e-05	-0.35	0.0038

Note. Figures in parentheses are standard errors.

Table 4: Unrestricted Estimates of α and β^*

First, Table 4 presents a set of unrestricted estimates, where $\widehat{\beta}^*$ is normalised for $p_t - w_t$. Since no restriction is imposed, the cointegrating space is not identified and no standard error can be reported for $\widehat{\beta}^*$. Uncertainty involved in estimation is all absorbed in $\widehat{\alpha}$, which is rather difficult to evaluate or interpret at it stands. It is therefore necessary to restrict $\widehat{\alpha}$ and $\widehat{\beta}^*$ in such a way that we can draw some inference about their significance and economic implications.

As demonstrated in Section 2, weak exogeneity plays an important role in the model reduction procedure. Testing weak exogeneity in the $I(1)$ cointegrated system corresponds to checking zero restrictions on elements of α. If rs_t and Δy_t are judged to be weakly exogenous for parameters of interest, one has only to model the conditional system for the variables $p_t - w_t$ and Δp_t in order to conduct inferences with no loss of information. With regard to restrictions on β, it is of interest to identify the long-run empirical relationship interpreted as the markup equation given by (3). It is expected that the inflation process could play a little or no role in the long-run relationship, so the exclusion of Δp_t from the long-run cointegrating relation should be examined. Thus, the following restrictions on α and β^* are jointly investigated:

1. zero restrictions on the elements of α corresponding to rs_t and Δy_t,
2. a zero restriction on the element of β^* corresponding to Δp_t.

A set of restricted estimates is given in Table 5, together with the corresponding log LR test statistic and p-value. Table 5 shows that the set of hypotheses is not rejected at the

	$p_t - w_t$	Δp_t	Δy_t	rs_t	t	$\chi^2(3)$
$\widehat{\alpha}'$	-0.19	0.19	0	0	$-$	4.76[0.19]
	(0.05)	(0.05)	$(-)$	$(-)$		
$\widehat{\beta}^{*\prime}$	1	0	1.12	0.002	0.00389	
	$(-)$	$(-)$	(0.51)	(0.004)	(1.51e-04)	

Note. Figures in parentheses are standard errors.

Table 5: Joint Restrictions on α and β^*

5% level, indicating that rs_t and Δy_t are weakly exogenous for the parameters of interest and Δp_t can be excluded from the cointegrating space. It turns out that the coefficient for rs_t in the cointegrating space is insignificant and that for Δy_t is close to unity. Thus, it would be worthwhile to test additional restrictions as follows:

1. a zero restriction on the element of β^* corresponding to rs_t,
2. a unitary restriction on the element of β^* corresponding to Δy_t.

Table 6 presents a set of restricted estimates, together with the corresponding log LR test statistic and p-value. Again, the set of hypotheses is not rejected at the 5% level,

	$p_t - w_t$	Δp_t	Δy_t	rs_t	t	$\chi^2(5)$
$\widehat{\alpha}'$	-0.18	0.19	0	0	$-$	5.01[0.41]
	(0.04)	(0.04)	$(-)$	$(-)$		
$\widehat{\beta}^{*\prime}$	1	0	1	0	0.00381	
	$(-)$	$(-)$	$(-)$	$(-)$	(7.15e-05)	

Table 6: Further Restrictions on α and β^*

indicating that the long-run economic relationship is given by

$$p_t - (w_t - 0.00381t) + \Delta y_t, \tag{10}$$

Interpreting the linear trend as an approximation of labour productivity growth, this cointegrating relation indicates that a markup over productivity-adjusted wages tends to move in the opposite direction to the real output growth. Relationship (10) is therefore interpreted as (3), consistent with the accepted view of *countercyclical markup*, discussed in the literature of macro and labour economics. See Blanchard and Fisher, 1989, Ch.9; Solon, Barsky and Parker, 1994; Rotemberg and Woodford, 1999; Romer, 2001, Ch.5, *inter alia*. Various theories have been developed to explain this pattern of countercyclical markup such as collusion in imperfect competition and kinked demand curves, see the above references. Under these economic interpretations, (10) can be justified as the representation of a meaningful long-run economic relationship.

3.5 An Equilibrium Correction Model

We are now in a position to achieve an equilibrium correction model for Japan's markup and inflation, corresponding to the bivariate system (4) in Section 2. The concept of productivity-adjusted wages is used to provide appropriate empirical markup, that is, an adjusted wage index is defined as $w_t^* = w_t - 0.00381t$ such that markup is represented by $p_t - w_t^*$. As a result of this adjustment, the sample mean of $\Delta\left(p_t - w_t^*\right)$ appears to be around zero rather than a negative value.

The starting point of the analysis is to map the data to the $I(0)$ space by differencing and using the restricted cointegrating combination. We then estimate a two-dimensional $I(0)$ VAR system for $\Delta\left(p_t - w_t^*\right)$ and $\Delta^2 p_t$ conditional on Δrs_t and $\Delta^2 y_t$. A set of insignificant terms, $\Delta^2 p_{t-2}$, $\Delta^2 p_{t-3}$, $\Delta^2 y_{t-1}$, $\Delta^2 y_{t-2}$ and Δrs_{t-3}, are dropped so as to reach a parsimonious VAR model. Terms with large standard errors then continue to be removed, that is, $\Delta\left(p_{t-2} - w_{t-2}^*\right)$ from the equation for $\Delta\left(p_t - w_t^*\right)$, and Δrs_{t-1} and Δrs_{t-2} from the $\Delta^2 p_t$ equation. Imposing constraints on some of the coefficients which have similar size in order to seek a parsimonious representation, an empirical price-wage mechanism is attained in (11).

$$
\begin{aligned}
\Delta\left(\widehat{p_t - w_t^*}\right) &= \underset{(0.05)}{-0.12}\,\Delta^a rs_t - \underset{(0.02)}{0.08}\Delta^{2a} y_t + \underset{(0.09)}{0.27}\Delta(p_{t-1} - w_{t-1}^*) - \underset{(0.08)}{0.14}\Delta^2 p_{t-1} \\
&\quad - \underset{(0.06)}{0.19}\Delta rs_{t-1} + \underset{(0.07)}{0.11}\Delta\left(p_{t-3} - w_{t-3}^*\right) - \underset{(0.03)}{0.17} ecm_{t-1} + \underset{(0.01)}{0.05}, \\
&\quad \widehat{\sigma} = 0.0038,\ F_{ar}(6{,}81) = 1.97[0.08], \\
&\quad \chi^2_{nd}(2) = 2.75[0.25],\ F_{arch}(6{,}78) = 0.35[0.91],\ F_{het}(20{,}69) = 1.02[0.46], \\
\Delta^2 \widehat{p}_t &= -\underset{(0.06)}{0.12}\Delta rs_t - \underset{(0.03)}{0.09}\Delta^2 y_t - \underset{(0.09)}{0.64}\Delta^2 p_{t-1} - \underset{(0.07)}{0.17}\Delta^a(p_{t-1} - w_{t-1}^*) \\
&\quad + \underset{(0.08)}{0.4}\,\Delta(p_{t-2} - w_{t-2}^*) - \underset{(0.03)}{0.16} ecm_{t-1} + \underset{(0.01)}{0.05}, \\
&\quad \widehat{\sigma} = 0.0038,\ F_{ar}(6{,}81) = 2.18[0.053], \\
&\quad \chi^2_{nd}(2) = 2.73[0.26],\ F_{arch}(6{,}78) = 0.33[0.92],\ F_{het}(20{,}69) = 1.02[0.45],
\end{aligned}
\tag{11}
$$

$$Vector\ Tests: F_{ar}(24{,}154) = 1.18[0.27],\ \chi^2_{nd}(4) = 4.69[0.32],\ F_{het}(60{,}200) = 0.90[0.68],$$

where

$$ecm_t = p_t - w_t^* + \Delta y_t,$$

and

$$
\begin{gathered}
\Delta^a rs_t = \Delta rs_t + \Delta rs_{t-2},\ \Delta^{2a} y_t = \Delta^2 y_t + \Delta^2 y_{t-3}, \\
\Delta^a(p_{t-1} - w_{t-1}^*) = \Delta(p_{t-1} - w_{t-1}^*) - \Delta(p_{t-3} - w_{t-3}^*).
\end{gathered}
$$

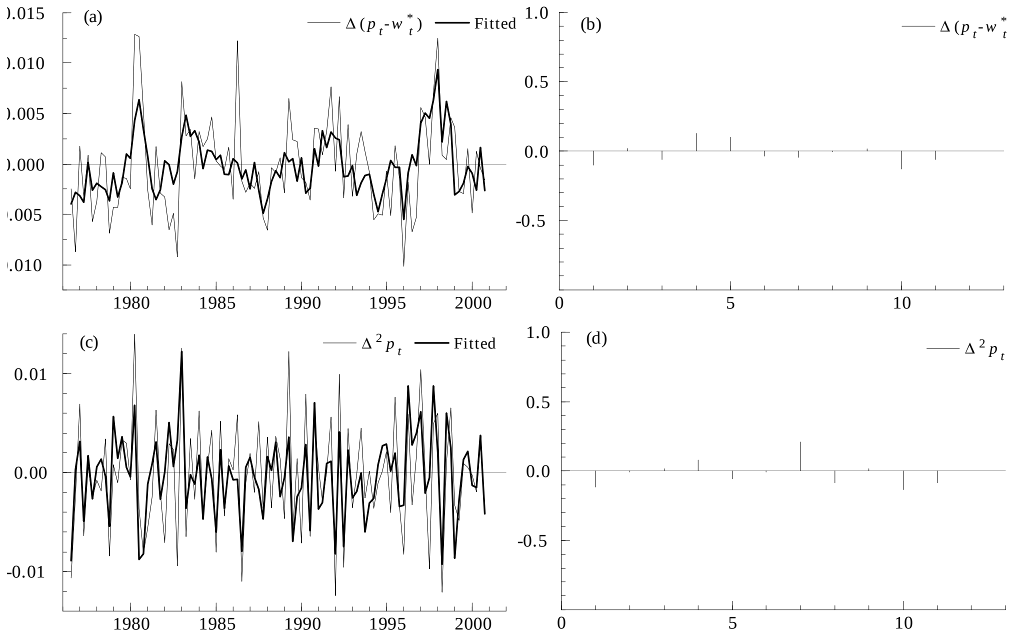

Figure 3: Fitted and Actual Values, Residual Autocorrelations

The method of constrained full-information maximum likelihood was used to estimate (11). The interest rate differential is divided by 100 to avoid reporting small coefficients. The standard errors on coefficients are given in parentheses. The test statistics for a single equation are reported under each equation, and the statistics for the whole system (the vector tests) are reported under the two equations. None of the diagnostic test statistics are significant, suggesting that the parsimonious system is a data-congruent representation. Figure 3(a)(c) record the actual and fitted values, and Figure 3(b)(d) show correlogram for each equation. Figure 4(a)(b) display the scaled residuals, Figure 4(c)(d) show recursive 1-step residuals, and break-point Chow tests are presented in 4(e)(f) (see Chow, 1960). None of the graphs indicates evidence against congruency. A data-congruent empirical model of a price-wage mechanism in Japan has been successfully estimated over two decades covering the 1980s and 1990s, a period of the pre and post Japan's bubble economy. It is noteworthy that such a stable structure has been revealed from the analysis of the data covering the period of Japan's economic turmoil.

Next, let us consider interpretation of the parsimonious system (11). A change in the interest rate differential has a negative effect on the markup and inflation growth; this may reflect information on expected economic growth and inflation contained in the differential. In line with the interest rate differential, an acceleration of the real output growth also has a negative influence on the markup and inflation growth. This is consistent with the countercyclical behaviour found in the cointegrating relation, and could be interpreted as a short-run reflection of this. As expected, the equilibrium correction term has a significant negative coefficient in both equations, indicating a stable adjustment toward the long-run equilibrium.

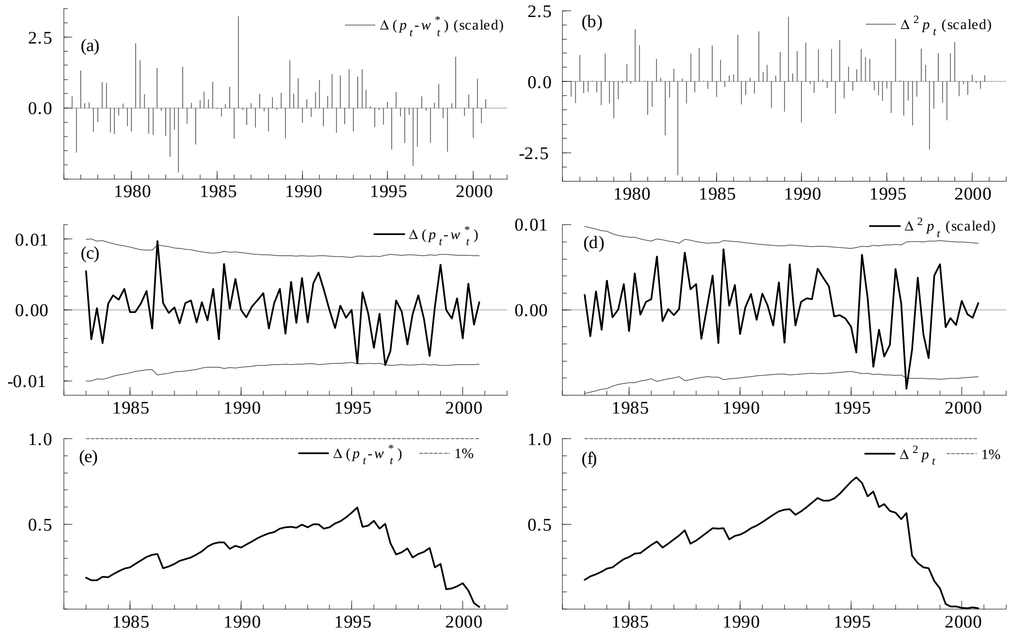

Figure 4: Scaled and Recursive Residuals, Break-Point Chow Tests

4 Summary and Conclusion

This paper, using a cointegrated VAR methodology, estimates a data-congruent econometric model for Japan's markup and inflation. The analysis provides evidence for the presence of a long-run economic relationship in the data, interpreted as an empirical representation of countercyclical markup. A set of variables in the cointegrated system except markup and inflation are judged to be weakly exogenous for parameters of interest, thereby enabling us to estimate a partial system given the weakly exogenous variables. The model reduction is then conducted in order to attains a parsimonious dynamic econometric system. The preferred parsimonious system has passed a battery of diagnostic tests, thereby being judged to be a data-congruent representation of countercyclical markup and inflation dynamics. It should be noted that such a stable model has been estimated from the analysis of the data covering the period of Japan's economic turmoil. The empirical exploration sheds useful light on deeper understanding of the Japanese economy in the last quarter of the 20th century.

Appendix:

A Details of Weak Exogeneity

This section, based on Engle *et al.* (1983), reviews the concept of weak exogeneity. Suppose that the density function for the process X_t in Section 2 is given by $D_X(X_t|\mathbf{X}_{t-1};\boldsymbol{\lambda})$, where $\mathbf{X}_{t-1}$ represents past information, *i.e.* $\mathbf{X}_{t-1} = (X_{-k+1}, \ldots, X_{t-1})$, and $\boldsymbol{\lambda}$ is a set of parameters that belong to the parameter space $\boldsymbol{\Lambda}$. Let $\boldsymbol{\lambda}$ be partitioned in $(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)$ to support the factorization of $D_X(X_t|\mathbf{X}_{t-1};\boldsymbol{\lambda})$ into the conditional and the marginal density functions, in line with $X_t = (X_{1t}', X_{2t}')'$ in Section 2, that is,

$$D_X(X_t|\mathbf{X}_{t-1};\boldsymbol{\lambda}) = D_{X_1|X_2}(X_{1t}|X_{2t}, \mathbf{X}_{t-1};\boldsymbol{\lambda}_1) \cdot D_{X_2}(X_{2t}, \mathbf{X}_{t-1};\boldsymbol{\lambda}_2).$$

Suppose that the parameters of interest coincide with $\boldsymbol{\lambda}_1$, and X_{2t} is then said to be weakly exogenous for $\boldsymbol{\lambda}_1$ if $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ are variation free, that is,

$$(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) \in \boldsymbol{\Lambda}_1 \times \boldsymbol{\Lambda}_2, \quad \text{where } \boldsymbol{\lambda}_1 \in \boldsymbol{\Lambda}_1 \text{ and } \boldsymbol{\lambda}_1 \in \boldsymbol{\Lambda}_2.$$

Thus $\boldsymbol{\Lambda}$ is the direct product of $\boldsymbol{\Lambda}_1$ and $\boldsymbol{\Lambda}_2$. In the context of the VAR model in Section 2, if $\alpha_2 = 0$ holds, the parameters in (9) coincide with $\boldsymbol{\lambda}_1$ such that the variation free condition given above is fulfilled. It is therefore concluded that X_{2t} is weakly exogenous for (9) in this case. If, however, $\alpha_2 = 0$ does not hold, the cointegrating parameter β then enters both $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ such that β acts as a cross-equation restriction, thereby violating the variation free condition.

B Details of the Data

(Data Definitions)

$p_t - w_t =$ the log of the implicit deflator for GDP in Japan, <1>
$-$ the log of monthly earning index in Japan, <2>,
$\Delta p_t =$ the first-order difference of p_t, <1>,
$y_t =$ the first-order difference of the log of the real GDP in Japan, <1>,
$rs_t =$ the government bond yield minus the deposit rate in Japan, <2>

(Sources and Notes)

<1> *System of National Accounts (68SNA)*, Japanese Cabinet Office web page.

<2> *International Financial Statistics*, International Monetary Fund.

The implicit deflator is constructed from the division of the nominal GDP by the real GDP. The nominal and real GDP seasonally-adjusted series, so that the implicit deflator is seasonally adjusted. The monthly earning index is also seasonally adjusted by the X12 ARIMA procedure.

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